Compiler Design
Lexical Analysis
Optimization of DFA-Based Pattern Matchers

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Outline

- Important States of an NFA
- Functions Computed from the Syntax Tree
- Computing *nullable*, *firstpos* and *lastpos*
- Computing *followpos*
- Converting a Regular Expression Directly to a DFA
- Minimizing the Number of States of a DFA
- State Minimization of a Lexical Analyzers
- Trading Time for Space in DFA Simulation

Optimization of DFA-Based Pattern Matchers

- First algorithm
  - constructs a DFA directly from a regular expression
  - without constructing an intermediate NFA
  - with fewer states
  - used in Lex
- Second algorithm
  - minimizes the number of states of any DFA
  - combines states having the same future behavior
  - has O(n*log(n)) efficiency
- Third algorithm
  - produces more compact representations of transitions tables then the standard two dimensional ones

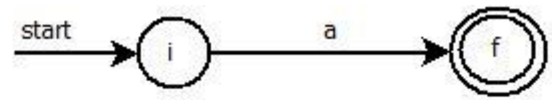
Important States of an NFA

- it has non-ε out transitions
- used when computing *ε-closure(move(T,a))* – the set of states reachable from *T* on input *a*
- the set *moves(s,a)* is non-empty if state *s* is important

- NFA states are twofold if
  - have the same important states, and
  - either both have accepting states or neither does

Augmented Regular Expression

- important states
  - initial states in the basis part for a particular symbol position in the RE

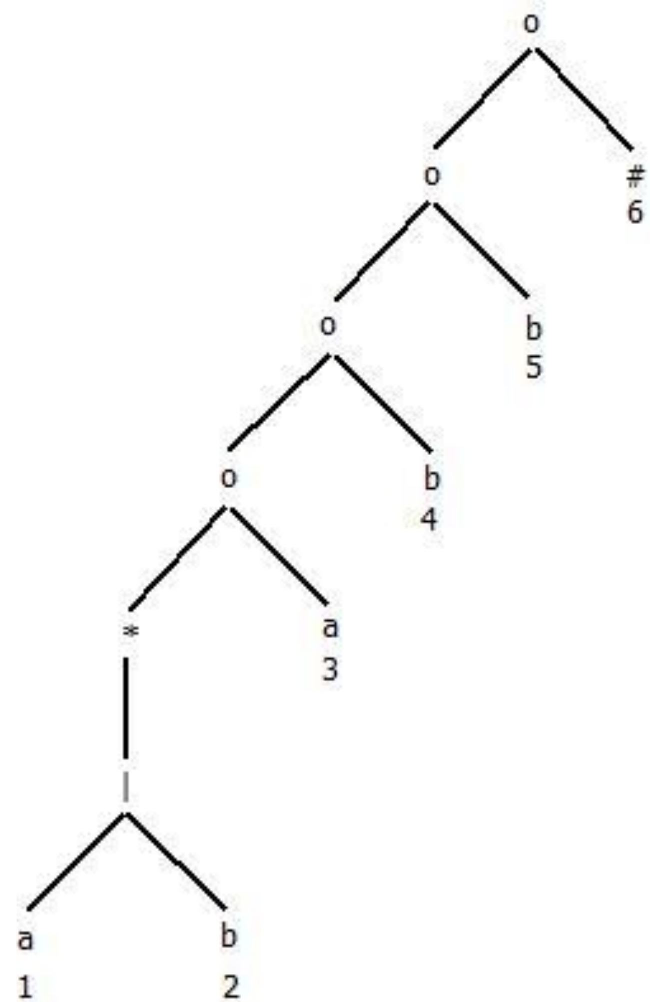


  - correspond to particular operands in the RE
- Thompson algorithm constructed NFA
  - has only one accepting state which is non-important (has no out-transitions !!!)
- to concatenate a unique right endmarker # to a regular expression r
  - the accepting state of the NFA r becomes important state in the (r)# NFA
  - any state in the (r)# NFA with a transition to # must be an accepting state

Syntax Tree

- important states correspond to the positions in the RE that hold symbols of the alphabet
- RE representation as syntax tree
  - leaves correspond to operands
  - interior nodes correspond to operators
    - cat-node – concatenation operator (dot)
    - or-node – union operator |
    - star-node – star operator *

Syntax Tree Example (a|b)*abb#

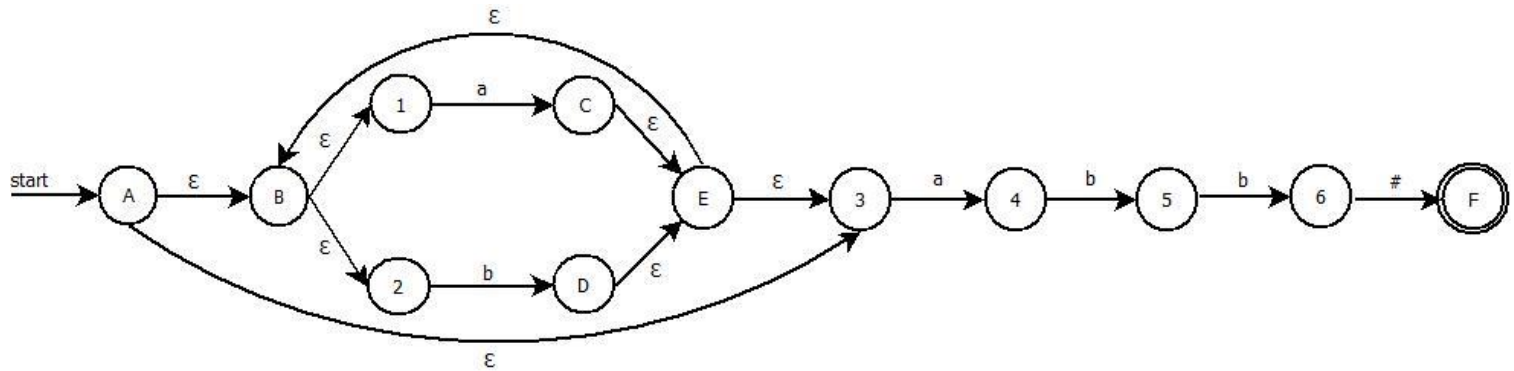


cat nodes are represented as circles

Representation Rules

- syntax tree leaves are labeled by ε or by an alphabet symbol
- to each leaf which is not ε we attach a unique integer
  - the position of the leaf
  - the position of it's symbol
- a symbol may have several positions
  - symbol *a* has positions 1 and 3 (on the next slide!!!)
- positions in the syntax tree correspond to NFA important states

Thompson Constructed NFA for (a|b)*abb#



- important states are numbered
- other states are represented by letters
- the correspondence between
    - numbered states in the NFA and
    - the positions in the syntax tree
- will be presented next

Functions Computed from the Syntax Tree

- in order to construct a DFA directly from the regular expression we have to:
  - build the syntax tree
  - compute 4 functions referring (r)#
    - nullable
    - firstpos
    - lastpost
    - followpos

Computed Functions

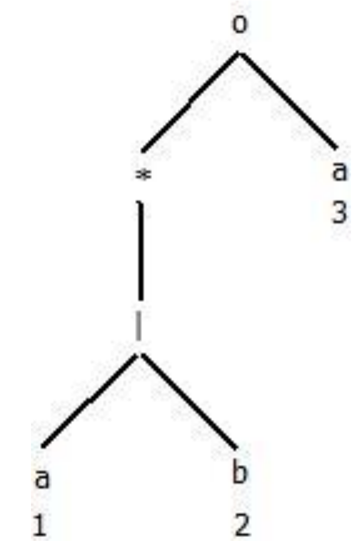
- nullable(n)
  - true for syntax tree node n iff the subexpression represented by n
    - has ε in its language
    - can be made null or the empty string even it can represent other strings
- firstpos(n)
  - set of positions in the n rooted subtree that correspond to the first symbol of at least one string in the language of the subexpression rooted at n

Computed Functions

- lastpos(n)
  - set of positions in the n rooted subtree that correspond to the last symbol of at least one string in the language of the subexpression rooted at n
- followpos(n)
  - for a position p
  - is the set of positions q such that
  - $x=a_1a_2\ldots a_n$ in L((r)#) such that
  - for some i there is a way to explain the membership of x in L((r)#) by matching a_i to position p of the syntax tree a_{i+1} to position q

Example

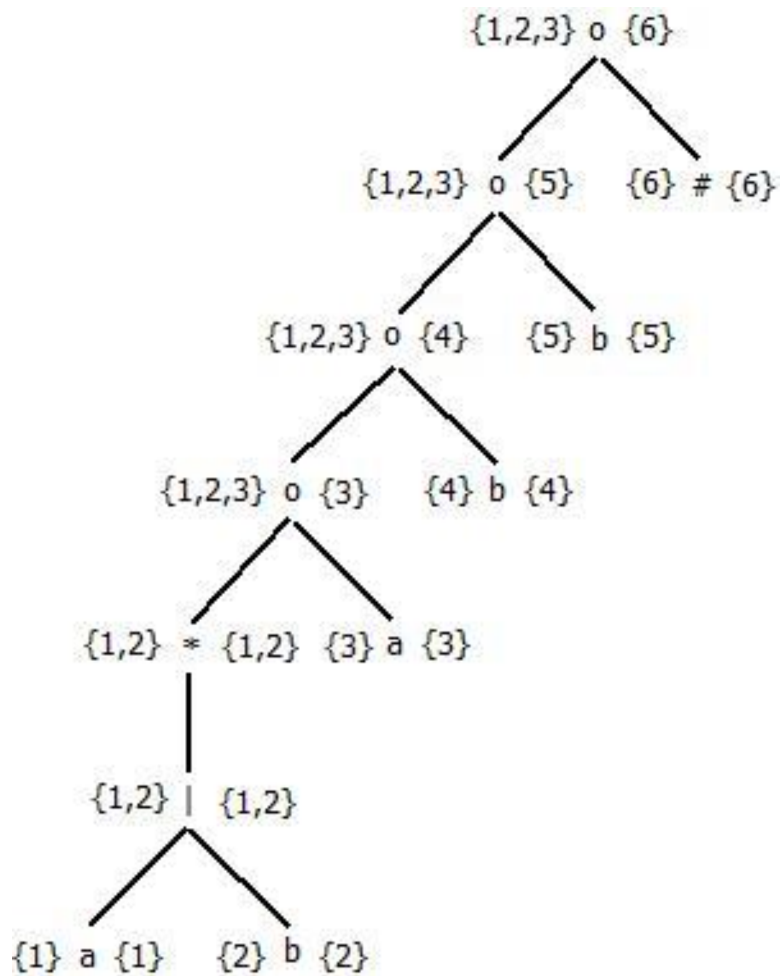
- nullable(n)=false
- firstpos(n)={1,2,3}
- lastpos(n)={3}
- followpos(1)={1,2,3}



Computing nullable, firstpos and lastpos

node n	nullable(n)	firstpos(n)	lastpos(n)
A leaf labeled ε	true	ø	ø
A leaf with position i	false	{i}	{i}
An or-node $n=c_1\|c_2$	nullable(c1) or nullable(c2)	firstpos(c_1) U firstpos(c_2)	lastpos(c_1) U lastpos(c_2)
A cat-node $n=c_1c_2$	nullable(c1) and nullable(c2)	if (nullable(c_1)) firstpos(c_1) U firstpos(c_2) else firstpos(c_1)	if (nullable(c_2)) lastpos(c_2) U lastpos(c_1) else lastpos(c_2)
A star-node $n=c_1$*	true	firstpos(c_1)	lastpos(c_1)

Firstpos and Lastpos Example

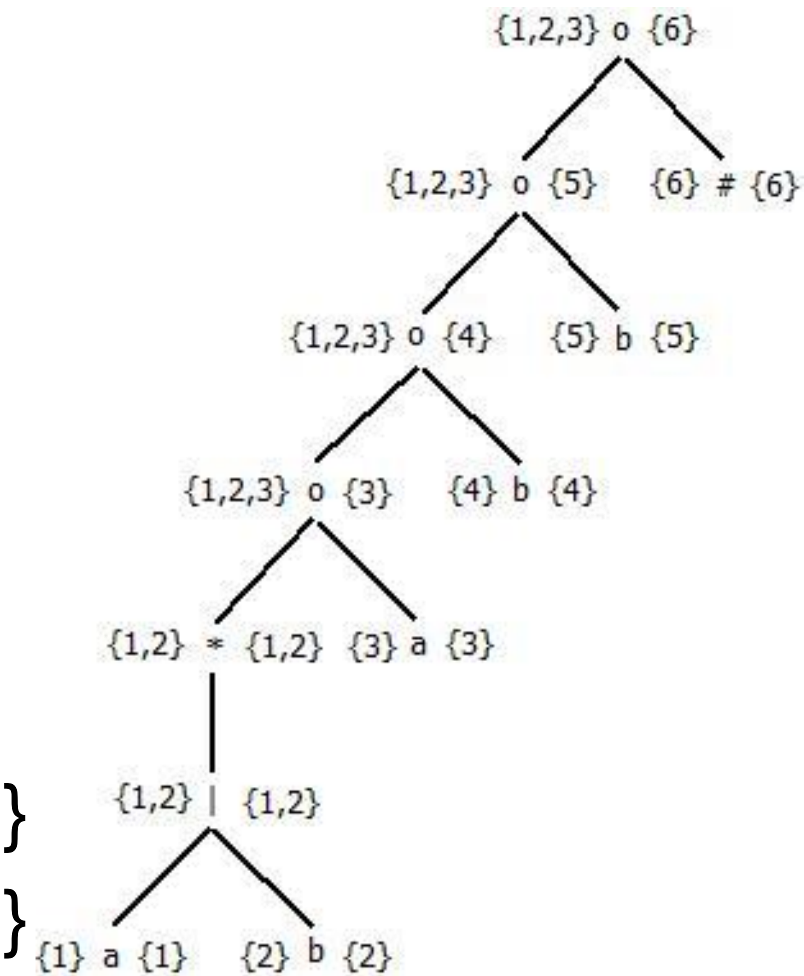


Computing Followpos

- A position of a regular expression can follow another position in two ways:
  - if n is a cat-node c_1c_2 (rule 1)
    - for every position i in lastpos(c_1) all positions in firstpos(c_2) are in followpos(i)
  - if n is a star-node (rule 2)
    - if i is a position in lastpos(n) then all positions in firstpos(n) are in followpos(i)

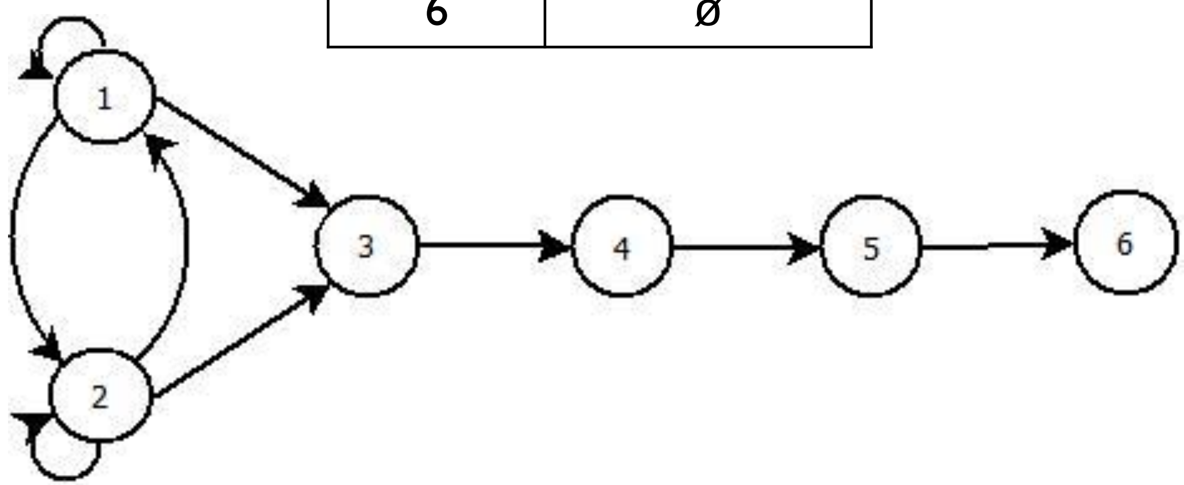
Followpos Example

- Applying rule 1
  - followpos(1) incl. {3}
  - followpos(2) incl. {3}
  - followpos(3) incl. {4}
  - followpos(4) incl. {5}
  - followpos(5) incl. {6}
- Applying rule 2
  - followpos(1) incl. {1,2}
  - followpos(2) incl. {1,2}



Followpos Example Continued

Node n	followpos(n)
1	{1,2,3}
2	{1,2,3}
3	{4}
4	{5}
5	{6}
6	ø



Converting a Regular Expression Directly to a DFA

- Input
  - a regular expression r
- Output
  - A DFA D that recognizes L(r)
- Method
  - to build the syntax tree T from (r)#
  - to compute nullable, firstpos, lastpos, followpos
  - to build
    - Dstates the set of DFA states
      - start state of D is firstpos(n_0), where n_0 is the root of T
      - accepting states = those containing the # endmarker symbol
    - Dtran the transition function for D

Construction of a DFA directly from a Regular Expression

```
initialize Dstates to contain only the unmarked
  state firstpos(n0), where n0 is the root of
  syntax tree T for (r)#;
while(there is an unmarked state S in Dstates)
{
  mark S;
  for(each input symbol a)
  {
      let U be the union of followpos(p) for all
      p in S that correspond to a;
      if(U is not in Dstates)
            add U as unmarked state to Dstates;
      Dtran[S,a]=U;
  }
}
```

Example for r=(a|b)*abb

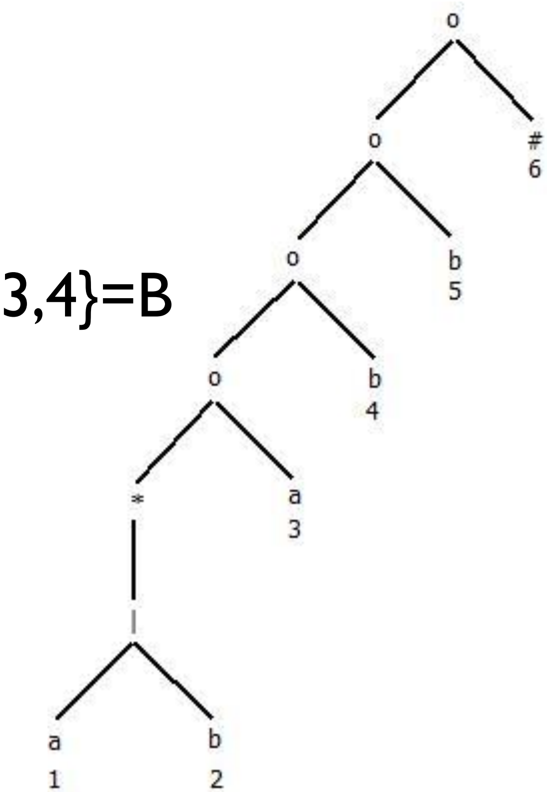
- A=firstpos(n_0)={1,2,3}
- Dtran[A,a]=

  followpos(1) U followpos(3)= {1,2,3,4}=B
- Dtran[A,b]=

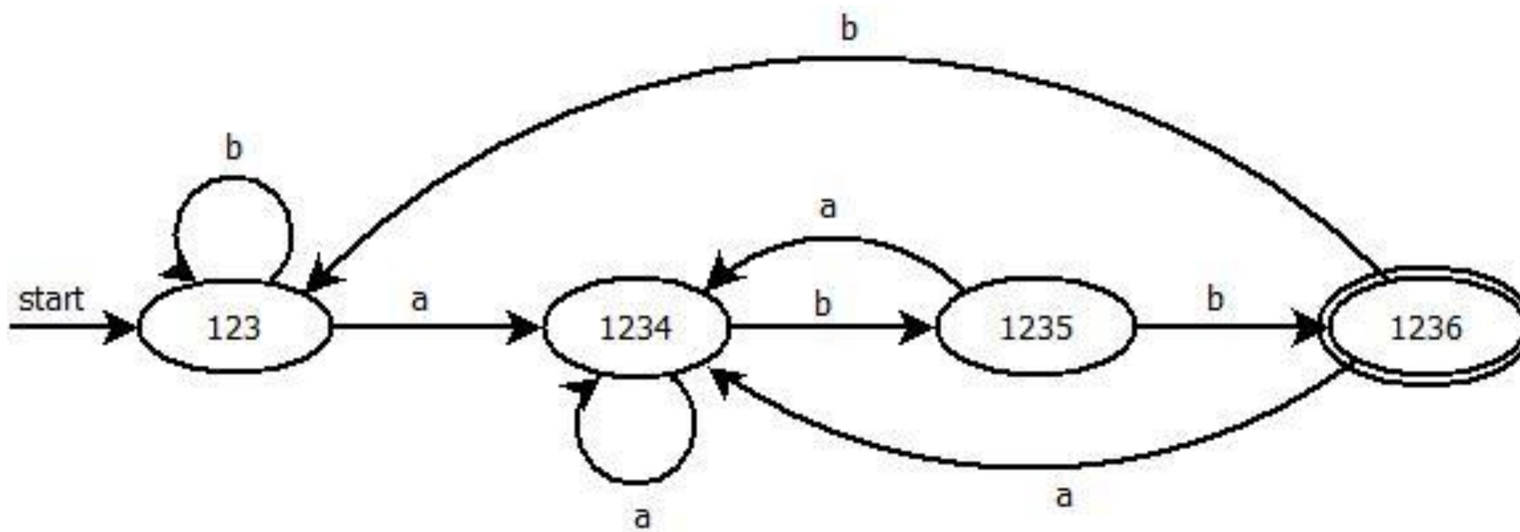
  followpos(2)={1,2,3}=A
- Dtran[B,a]=

  followpos(1) U followpos(3)=B
- Dtran[B,b]=

  followpos(2) U followpos(4)={1,2,3,5}=C
- …

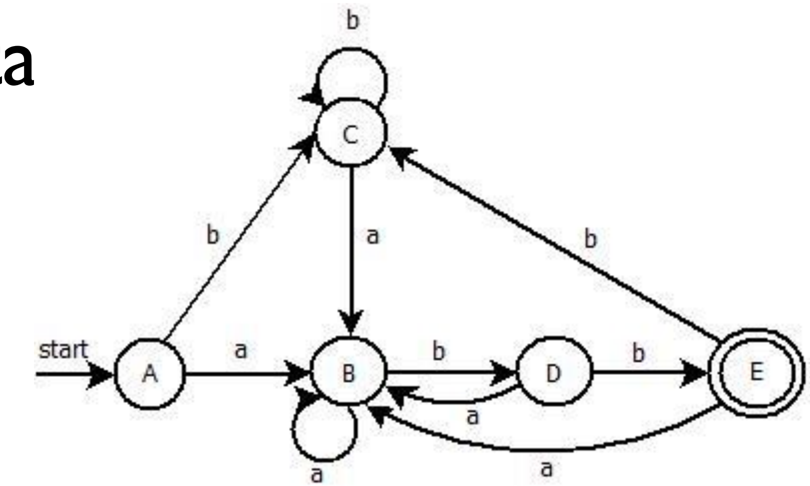


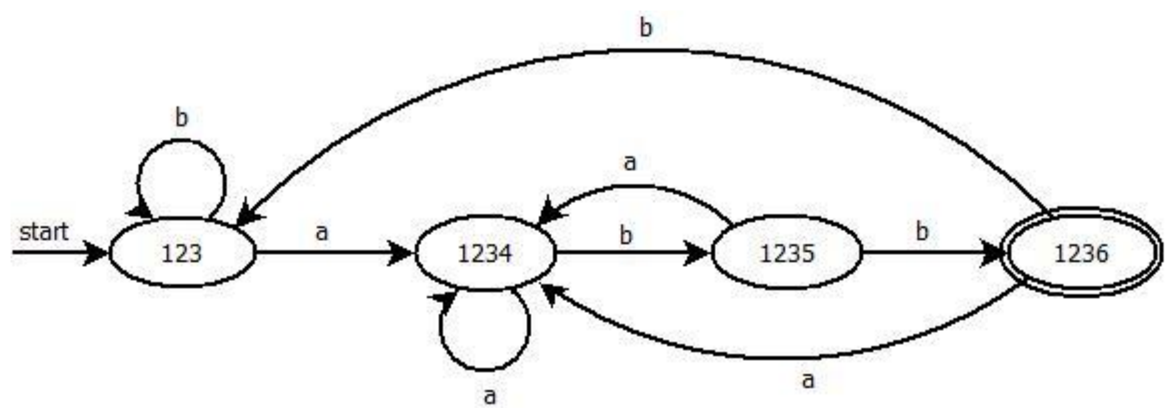
Example for r=(a|b)*abb



Minimizing the Number of States of a DFA

- equivalent automata
  - {A,C}=123
  - {B}=1234
  - {D}=1235
  - {E}=1236
- exists a minimum state DFA !!!





Distinguishable States

- string x distinguishes state s from state t if exactly one of the states reached from s and t by following the path x is an accepting state
- state s is distinguishable from state t if exists some string that distinguish them
- the empty string distinguishes any accepting state from any non-accepting state

Minimizing the Number of States of a DFA

- Input
  - DFA D with set of states S, input alphabet Σ, start state s_0, accepting states F
- Output
  - DFA D' accepting the same language as D and having as few states as possible

Minimizing the Number of States of a DFA

1 Start with an initial partition Π with two groups F and S-F

2 Apply the procedure

for(each group G of Π)

{

partition G into subgroups such that states s and t are in the same subgroup iff for all input symbol a states s and t have transitions on a to states in the same group of Π

}

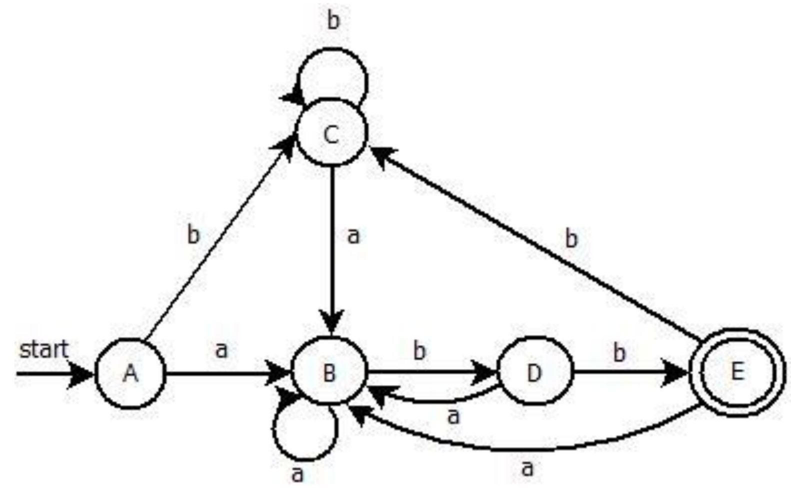
3 if $\Pi_{new} = \Pi$ let $\Pi_{final} = \Pi$ and continue with step 4, otherwise repeat step 2 with Π_{new} instead of Π

4 choose one state in each group of Π_{final} as the representative for that group

Minimum State DFA Construction

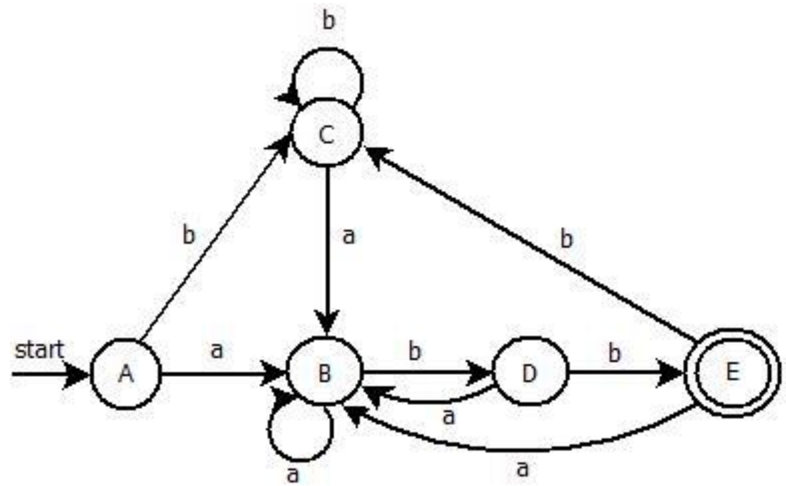
- the start state of D' is the representative of the group containing the start state of D
- the accepting states of D' are the representatives of those groups that contain an accepting state of D
- if
  - s is the representative of G from Π_{final}
  - exists a transition from s on input a is t from group H
  - r is the representative of H
- then
  - in D' there is a transition from s to r on input a

Example



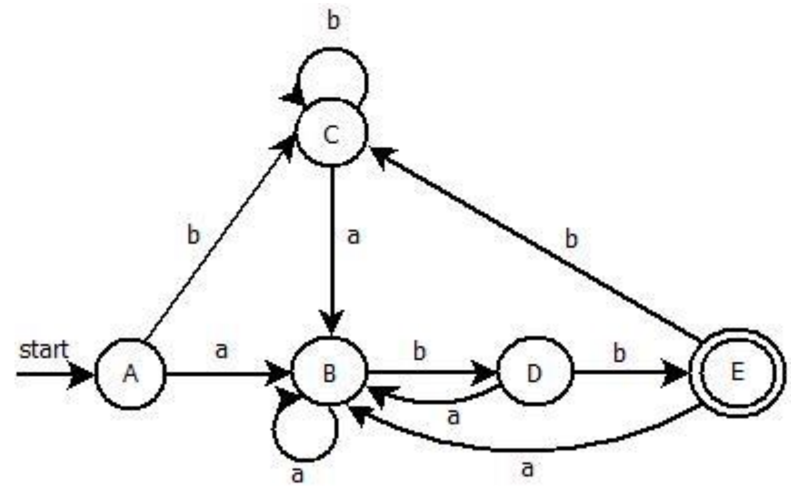
- {A,B,C,D}{E}
  - on input a:
    - A,B,C,D->{A,B,C,D}
    - E->{A,B,C,D}
  - on input b:
    - A,B,C->{A,B,C,D}
    - D->{E}
    - E->{A,B,C,D}

Example



- {A,B,C}{D}{E}
  - on input a:
    - A,B,C->{A,B,C}
    - D->{A,B,C}
    - E->{A,BC}
  - on input b:
    - A,C,->{A,B,C}
    - B->{D}
    - D->{E}
    - E->{A,B,C}

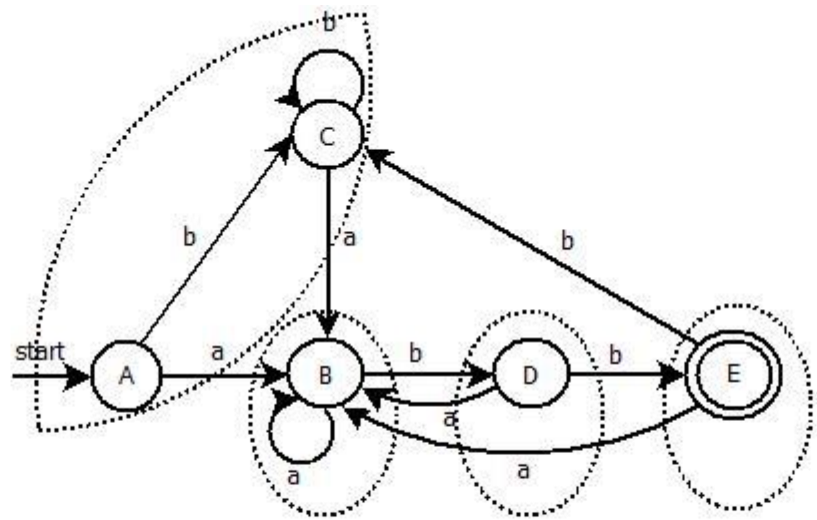
Example



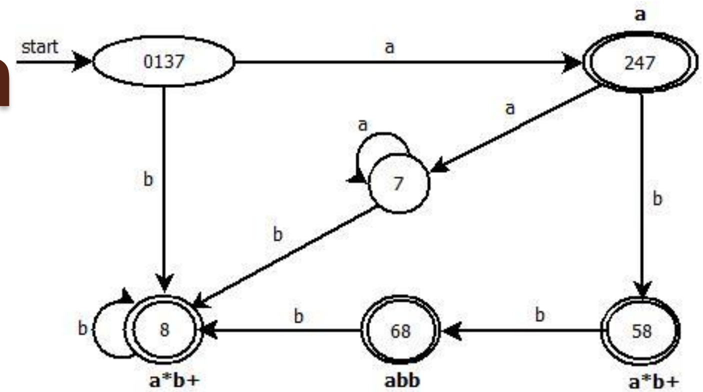
- {AC}{B}{D}{E}
  - on input a:
    - A,C->{B}
    - B->{B}
    - D->{B}
    - E->{B}
  - on input b:
    - A,C,->{A,C}
    - B->{D}
    - D->{E}
    - E->{A,C}

Example

State	a	b
A	B	A
B	B	D
D	B	E
E	B	A

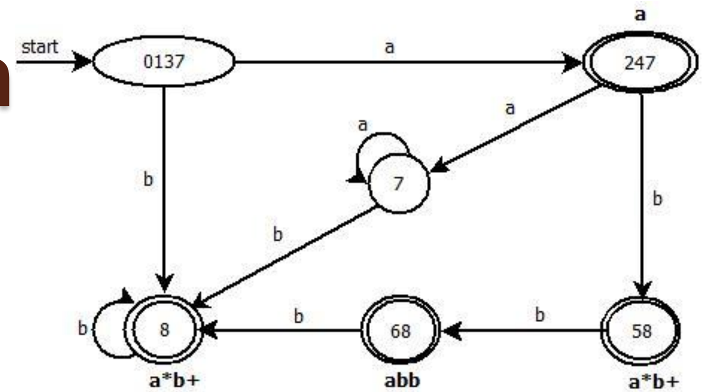


State Minimization in Lexical Analyzers



- to group together
  - all states that recognize a particular token
  - all states that do not indicate any token
- e.g. {0137,7} {247} {8,58} {7} {68} {ø}
  - {0137,7} – do not indicate any token
  - {8,58} – announce a*b+
  - {ø} - dead state
    - has transitions to itself on input a and b
    - is target state for states 8, 58, 68 on input a

State Minimization in Lexical Analyzers



- next, we split
  - 0137 from 7
    - they go to different groups on input a
  - 8 from 58
    - they go to different groups on input b
- dead states can be dropped
  - if we treat missing transitions as signal to end token recognition

Trading Time for Space in DFA Simulation

- transition function of a DFA
  - two dimensional table indexed by states and characters
- typical lexical analyzer has
  - hundreds of states
  - ASCII alphabet of 128 input characters
  - < 1 MB
- compilers "live" in small devices too
- 1 MB could be too much

Alternate Representations

- list of character-state pairs
- ending by a default state
  - chosen for any input character not on the list
  - the most frequently occurring next state
- thus, the table is reduced by a large factor

Bibliography

- Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman – Compilers, Principles, Techniques and Tools, Second Edition, 2007